Flows with Significant Orientational Effects

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I. Introduction

MOST of our understanding of fluid flows is based on a physical picture in which the elementary particles behave like smooth spheres. The fluid itself has no structure, and remains structureless when it flows in response to impressed external gradients. The force fields describing the interactions of the elementary particles are taken to have spherical symmetry. Angular momentum exchange is not allowed for in collisions between these particles, and the angular momentum vectors of the individual particles are assumed to be randomly oriented, so that their vector sum over any region is zero. The conventional Euler and Navier-Stokes equations, which express the mathematical consequences of this model, have provided an adequate basis for the solution of most fluid-mechanical problems for many years.

However, there are certain fluid-flow situations for which this model and its associated equations are inadequate: problems where the elementary particles themselves have a structure (such as the flow of macromolecules in solution), or where they take on a structure when set in motion (such as the alignment of turbulent eddies in an anisotropic flow); problems where the particles interact according to a noncentral force field (as in the kinetic theory of polyatomic gases), or where angular-momentum exchange occurs in a sufficiently organized way, so as to produce a nonzero value of the net angular momentum (such as occurs, under certain circumstances, in the flow of suspensions). For flows in which these phenomena arise, generalizations of the basic physical models and their associated equations of motion are required.

Many contributions to this field have been made during the past 10-20 years, so that there now exists a substantial literature which is capable of describing these phenomena. While these contributions have been reviewed on a number of occasions,[1-8] nevertheless they do not seem to be widely known. It was felt that there would be value in another survey of the field, written especially with aeronautical and aerospace interests in mind. It is hoped that the reader will acquire a somewhat broader vision of the mechanisms that may be at work in fluid-flow problems, and an initial indication of more general analyses that are available for solving some of these problems.

The viewpoint of this review is not to suggest that there are inadequacies in major portions of the contemporary fluid-mechanical literature that the new approaches reviewed here are intended to correct. Quite the contrary, the majority of the still-unsolved problems require only a continued application of the conventional theories, whose full scope is far from exhausted. This review is intended only to point out that, despite the vast room for further exploitation of traditional approaches, there are, nevertheless, some problems where a more detailed model of the flow is called for.

The paper consists of two main sections: the first contains a description of the basic physical phenomena embraced by these new fields, and the main features of the mathematical generalizations that accompany them. The second section presents a selected list of problem areas where the new phenomena may be expected to play a significant role.

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II. Basic Model

In the derivation of the conventional continuum equations for a fluid, the starting point is to consider the forces that can act on the faces of a "small" element of the fluid (see, for example, Ref. 9, Art. 1.1 and 1.2). The element (see Fig. 1) is always taken to be small in comparison with the scale of the problem being studied, but not so small that molecular effects are encountered. The faces of the element are pictured as planes across which subelemental particles* are continually passing in either direction. When such particles, coming from one side of a face, collide with those on the other side, an exchange of linear momentum takes place; the magnitude of this linear momentum exchange, per unit time, and per unit area of the face, is represented as a shear stress (force per unit area), and is written as a tensor

$$\tau = \begin{bmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{bmatrix}$$

It should be recognized that this model does not allow for the exchange of angular momentum during these collisions.

The new theories that are discussed here explicitly allow for this more general feature. Thus, the subelemental particles can be imagined as rotating disks, or as roughened spheres, so that they are also capable of transferring some of their *angular* momentum in their encounters with collision partners on the other side of the face. The magnitude of this angular momentum exchange, per unit time and unit area of the face, is represented as a couple stress (i.e., the couple moment per unit area), and is also written as a tensor (see Fig. 2).

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

*These subelemental particles may be atoms or molecules, or collections of these, or some larger aggregates, as in the case of turbulent eddies.

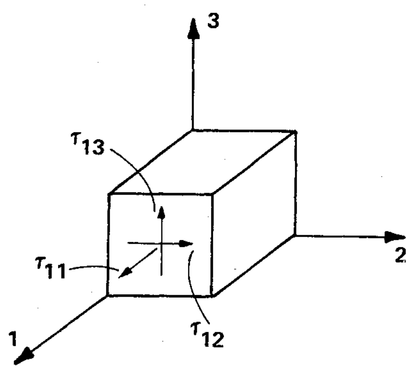

Fig. 1 Components of the shear-stress tensor.

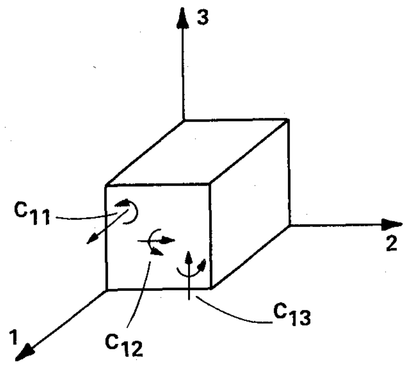

Fig. 2 Components of the couple-stress tensor.

This generalization is nothing more than the application of the familar principle that the forces acting on an element can always be represented by a net force plus a couple (see, for example, Ref. 10). The forces due to linear-momentum exchange are anticipated by the shear-stress tensor; the moments due to angular momentum exchange must also be allowed for, by postulating the existence of the couple-stress tensor.

The ability of the subelemental particles to exchange angular momentum is not enough, by itself, to guarantee a nonzero couple stress; it is also necessary that these exchanges occur in a sufficiently organized manner that their average value over the face of the element is nonzero. Thus, an assumption that the couple stresses are zero may safely be made if the angular velocity vectors of the subelemental particles are randomly distributed. In the same sense, the neglect of shear stresses in a conventional inviscid-fluid model does not imply that there are no particle collisions at all, but only that their net contribution to the linear momentum exchange across the faces of the element is zero.

The introduction of couple stresses has two further consequences: the shear-stress tensor can no longer be assumed symmetric, and allowance must be made for a nonzero "internal" angular momentum, which is the vector sum of the angular momenta of all the particles in the fluid element. In derivations of conventional fluid-mechanical equations, it is usually argued that the shear-stress tensor must be symmetric (i.e., $\tau_{ij}=\tau_{ji}$), since otherwise one would have a nonzero torque acting on an element of vanishing size in the limit where the dimensions of the element are shrunk toward zero.

This argument is made by writing, for the element, the fact that the net torque is equal to the time rate of change of the angular momentum. The net torque dT is proportional to the antisymmetric part of the shear stress $\Delta\tau$ (equal to $\tau_{12}-\tau_{21}$, for example), multiplied by the area of the face of the element [equal to $(ds)^2$, if the element is taken as a cube of volume $(ds)^3$], and then multiplied by a moment arm of order ds

$$dT \approx \Delta\tau \cdot (ds)^2 \cdot ds$$

The angular momentum dP is assumed to come only from solid-body rotation of the element as a whole; thus, it is equal to the moment of inertia of the element, multiplied by its angular velocity. The moment of inertia, in turn, is assumed to be proportional to the fifth power of the size of the element, i.e.

$$dP=\omega dI=\omega \cdot dm \cdot (ds)^2=\omega\rho(ds)^5$$

This expression is then equated to the above formula for the differential torque, and it is noted that the resulting equation cannot be satisfied when ds tends to zero, unless $\Delta\tau$ itself is assumed to vanish.

The key difference in the more general theory being reviewed here is that the element is allowed to contain an intrinsic angular momentum due to rotation of the subelemental particles, in addition to the angular momentum associated with solid-body rotation of the element as a whole. Thus, although the latter vanishes as $ds\rightarrow 0$, there remains an intrinsic angular momentum of the fluid within the element, which can be written as

$$dP=n(ds)^3 I_p \omega$$

where n is the number of particles per unit volume, and I_p and ω are their moment of inertia and angular velocity. When the substructure within the element is recognized in this way, the torque and angular momentum both vanish with the same power of ds, and it is no longer necessary to make any further assumption about the symmetry of the shear-stress tensor.

Thus, to summarize: in the conventional theory, the angular momentum (per unit mass), M, is just the vector product of the position vector r and the center-of-mass velocity u, the shear-stress tensor is symmetric, and there are no couple stresses

$$M=r\times u \quad \tau_{ij}=\tau_{ji} \quad C_{ij}\equiv 0$$

In the generalized theory discussed here, the angular momentum is supplemented by the internal contribution l, the shear-stress tensor is asymmetric, and the couple-stress tensor is nonzero

$$M=r\times u+l \quad \tau_{ij}\neq\tau_{ji} \quad C_{ij}\neq 0$$

The divergence of the couple-stress tensor, the antisymmetric part of the shear stress tensor, and a body couple G (per unit mass), which represents the moment due to a magnetic, gravitational, or other field that acts at a distance, contribute to the rate of change of the internal angular momentum:

$$\rho\frac{Dl}{Dt}=\text{div } C+\begin{bmatrix}\tau_{23}-\tau_{32}\\ \tau_{31}-\tau_{13}\\ \tau_{12}-\tau_{21}\end{bmatrix}+\rho G$$

A complete derivation of the equations can be found in Refs. 1-4, 11.

The solutions of these equations differ from their conventional counterparts by the presence of the internal angular momentum vector and the couple-stress tensor. The solution must contain distributions of these quantities in addition to the customary distribution of pressure, density, and velocities. The number of flow situations for which these solutions have been worked out is quite limited, although most of the classical situations (Poiseuille flow, Couette flow, etc.) have been solved.[7,8] The introduction of internal angular momentum and couple stress, and the transport coefficients which connect the latter tensor to linear and angular velocity gradients give rise to a large number of parameters, and a wide range of variations in the formulation of a given problem. Thus, it is difficult to draw any generalizations from the existing solutions about the effects that asymmetric stress will have on a given problem.

III. Applications

This section contains a listing of problems known to the author where the phenomena described previously are likely to appear. The list is not intended to be complete in any sense, but even this incomplete list will give some idea of the scope of the field, and the large number of situations in which these effects arise. In citing references, an attempt has been made to select the more general ones, whose perusal will lead the reader to more detailed works.

These applications have been grouped into four categories, in the hope of lending some order. Some of the categories constitute well-established branches of continuum mechanics, where the generalized equations of motion have been applied for many years. Other categories would be better described as embryonic, in the sense that the extent to which they are affected by organized angular momentum exchange has not yet been studied very thoroughly, although the presence of the effect is not in doubt.

A. Ordered Fluids

Many substances, which are formed as the liquid phase of a polymeric material or as solutions of long-chain molecules, exhibit strong orientational effects, such as optical activity or anisotropy in the apparent viscosity. General theories for treating the flows of such fluids have been available for many years.[2,12-15] In fact, the need to explain the motion of these substances seems to have given early impetus to studies in this area.

Liquid crystals make up a large subclass of ordered fluids. Because of their great technical importance, research on their properties has become a field in itself. References 16-19 provide a representative sample of papers in which the response of these materials to a variety of environments is discussed. No attempt will be made to review all the ramifications of this broad field. However, readers wishing to explore the fluid-mechanical behavior of these substances will find that the basic equations are essentially those outlined in Sec. II.

To conclude this first category on a speculative note, it is interesting to cite the area of large deformations of solids under intense loading as a possible ordered-fluid situation. There are many instances in which solids undergo an intense impulsive loading, such as occurs during collision with a high-speed particle. During the early stages of the subsequent deformation, the material often responds as though it were a fluid of zero strength; during the very late stages, it behaves like an elastic or perhaps plastic solid.[20] There is a possibility that the motion at some intermediate stage may exhibit the kinds of directional effects that are discussed here, especially if the material has some initial anisotropy (as would be the case, for example, with a composite armor material, or a layered medium, in the case of meteoroid impact on a planetary surface). These possibilities suggest a link to the vast literature of directional effects in solid mechanics (see, for example, the review by Herrmann.[21]

B. Kinetic Theory

The conventional Navier-Stokes equations can be derived from the kinetic theory of a gas whose molecules interact according to a central force field.[22] Strictly speaking, this approximation should be valid only for monatomic gases; the force fields of any other gaseous molecules have an angular dependence, so that their interactions are dependent on their relative orientations. Despite this theoretical limitation, the monatomic-gas model has produced many significant results in regimes that lie well beyond its limits of strict validity. In a general way, this success can be attributed to the fact the tendency of the intermolecular forces to produce a net orientation is relatively weak, compared with the strong tendency toward isotropy produced by the high collision frequency. Thus, the orientational effects discussed here are more likely to occur at very high density, where ordered intermolecular forces are more probable, or at very low density, where the collision frequency is low, or when the ordering effects are augmented, either by external fields (magnetic, electric, gravitational, etc.) or by strong gradients in the flow.

In situations where orientational effects are important, it is necessary to generalize the kinetic theory by allowing the distribution function to depend on variables of the orientation space. The rate of change of the distribution function due to collisions between particles must then be calculated, using some model for the orientation-dependent interparticle potential. There is obviously a great latitude to the amount of detail that can be included in this potential, and the results obtained by a number of workers (Refs. 23-32 are representative) reflect these differences. In general, however, the results of these studies are equivalent to the equations introduced in Sec. II: they are generalizations of the Navier-Stokes equations, which display asymmetric states of stress and nonzero values of the couple stress and internal angular momentum. The shear- and couple-stress tensors contain the terms associated with shear viscosity and bulk viscosity, and, in addition, a term that has been called vortex viscosity,[6] which determines the rate of equilibration between the internal angular velocity of the fluid and the vorticity. The results of these various studies, while leading to essentially the same conservation equations, differ at the point where the stress tensors are expressed in terms of various gradients, multipled by appropriate transport coefficients. These differences reflect the specific details that are accounted for in describing the collisions between the particles.

At the present time, there have been a number of experiments in which these orientational effects have been observed. In the high-density range, Dahler[5] has described the response of a polar liquid to a rotating electric field; at the low-density limit, the review article of Beenakker and McCourt[33] contains a discussion of some of the effects caused by an impressed magnetic or electric field.

Several other observations deal with orientational effects that were produced by purely gasdynamic means, without the agency of an external field. These include: anisotropy in the random translational motions of a rarefied gas that has passed through a strong expansion[34,35]; a net orientation of sodium-vapor dimers, which had also passed through a strong expansion[36]; and streaming birefringence of gases in the strong shear field between two concentric, contrarotating cylinders.[37] These observations might be summarized by stating that orientational effects have been observed in shear flows and expansion waves. They strongly suggest that comparable phenomena could be observed in a shock wave, whose velocity gradient is much larger than that present in the rotating cylinder appartus just cited.[37] Haight and Lundgren[38] have presented a kinetic-theory study of the shock-structure problem, using loaded spheres and spherocylinders to model the collision dynamics. Their results would be useful as a starting point in interpreting measurements of the orientation produced by shock waves in gases.

As a postcript to this discussion of shock-induced orientation, mention should be made of the occurrence of the same phemomenon in liquids and solids (although these observations are not, strictly speaking, examples of the dilute-gas kinetic theory). For example, flow birefringence in liquids can be produced in either a shear field (streaming birefringence) or in a field of plane waves (acoustic birefringence).[39,40] The high-intensity limit of these phenomena is observed in the electrical signals produced when a shock wave propagates through a solid dielectric.[41,42] These electrical signals can be explained as the result of a polarization produced by the passage of the shock front.[43] Once this shock-induced polarization has been postulated, the description of the resulting electrical signal follows very closely the related treatment of the transient response of a piezoelectric material to the passage of a stress wave.[44-48] However, the detailed mechanics of how the polarization is produced by the shock wave in the first place is still an open question. The same basic mechanism is at work in producing ultrasonic vibration potentials,[49,50] which result from differences in the responses of anions and cations to the passage of a pressure wave through an ionic solution. A strong shock wave can also render a transparent solid doubly refracting,[51] presumably as a result of the same orientation that accounts for the electrical signal.

To conclude this kinetic-theory category, it should be stressed that the generalized equations of motion have been derived both from a continuum-mechanical viewpoint, and also as the continuum limit of the kinetic theory of gases with noncentral force fields. Thus, the more general equations enjoy the same theoretical foundation as that underlying the Navier-Stokes equations; they should not be regarded as curiosities of tentative merit, but as advanced approximations which may be of value in certain of the flow situations faced by practicing aeronautical engineers.

C. Suspensions

Many fluids of great technical importance are suspensions of finely divided particles dispersed in a carrier fluid. Examples of such suspensions range all the way from blood to atmospheric aerosols. Many of the flow phenomena exhibited by these fluids can be adequately treated by conventional methods, provided that such quantities as density, velocity, and the like are properly averaged between the two phases. But in recent years, it has also become clear that orientational effects are a common feature of the interactions between the carrier fluid and the suspended particles (see, for example, Refs. 52-54). The generality of this result is due to the fact that very special circumstances are required to *avoid* the exchange of angular momentum between the carrier fluid and the suspended particles. When the suspension is sheared, for example, the application of a nonrandomly oriented torque to the particles is the rule rather than the exception, and the effect is enhanced when the centers of mass of the particles do not coincide with the points at which the fluid force effectively acts. Brenner[52] strongly suggests that a number of problems in the flow of suspensions, hitherto classified as "non-Newtonian," will soon find a more accurate explanation in terms of the more general theories mentioned here. Many of these problems tend to lie in the field of rheology, where the carrier fluid is usually a liquid and the concentration of suspended particles is high. Significant angular momentum exchange can also occur at the other end of the spectrum, however, where the carrier fluid is a gas and the concentration is relatively low. As an example, studies of the role of asymmetric stress in the flow of a dusty gas have recently begun to appear.[55]

The foregoing studies of angular momentum exchange in suspensions deal only with laminar flows. There are many other applications in which the flow becomes turbulent. As will be seen later, the turbulence itself can often exhibit orientational effects, even in the absence of suspended particles. Thus, the field of turbulent flows of suspensions is likely to be one whose complexities will require a thorough accounting for organized angular momentum exchange. The equations reviewed in Sec. II from the starting point for such a theory.

One of the most important of these applications is the drag reduction achieved by adding a small amount of material to a carrier fluid, sometimes called the Toms phenomenon.[56-60] This effect, which has always seemed attractive in the context of ship motion, has also been used to advantage in pipelines and fire-fighting equipment.[61] The occurrence of the effect is not limited to linear shear flows or pipe flows; it has also been observed in vortex flows.[62]

An important instance of laminar suspension flows where orientational effects play a major role is that in which the suspension is designed to couple strongly to an external magnetic or electric field.[63-66] The magnetic fluids[67-69] offer a specific example where the generalized continuum approach has led to new results. Specifically, Hall and Busenberg[70] developed a theory and compared it with certain experimental measurements.[71] Their theory was based on energy considerations, but made no direct allowance for orientational effects. Brenner pointed out in his review article[52] that it was not obvious whether their approach was sufficiently general to embrace all of the asymmetric-stress effects that might occur. In a subsequent application of the new theory,[72] he was able to confirm their results, and developed, in the process, a considerably more detailed picture of the stress distribution.

A gravitational field can also produce orientational effects, even in a suspension whose particles are unaffected by electric or magnetic forces.[73,74] Thus, the situation with suspensions is very much like that of gases with noncentral force fields: orientational effects may develop as part of the fluid response to impressed gradients, and may also be generated directly by external fields.

Finally, a special subcategory of the flow of suspensions that is particularly intriguing is the problem of separating mixtures which contain equal amounts of particles with a left- and right-handed screw sense. There are a number of substances (called stereoisomers, enantiomers, or optical isomers) which exist in either of two geometrical arrangements of the same atoms, having a roughly helical screw sense. When substances of this type are synthesized in living media, only one isomeric configuration appears. Many synthetic processes for their production exist, but are usually not stereospecific, i.e., they tend to yield equal amounts of left- and right-handed particles, which must then be separated to be useful in biochemical or pharmaceutical applications.[75] The known separation processes are often quite expensive; thus, there is a continuing interest in fluid-mechanical means for achieving the separation.[52,76] It is possible that a diligent application of the theories discussed here might lead to such a means. Unfortunately, however, the response of even an isolated helical particle to a fluid flow is a subject that has been addressed only recently.[77,78]

D. Turbulence

It is sometimes suggested that the generalized-continuum concepts of asymmetric stress and organized angular momentum exchange would be of value in describing certain aspects of turbulent flows. To the writer's knowledge, this suggestion has never been pursued to the point of comparison between clearly formulated theoretical expressions and properly designed experiments. While the applicability of generalized-continuum concepts to turbulence is still very much in doubt, it was, nevertheless, felt that a critical review of the topic would be of value. It should be understood that this final category represents a much more speculative application than those already discussed.

Many advances in the prediction of turbulent flows have been made during the past decade. An excellent summary of the state-of-the-art as of 1968 can be found in Ref. 79, and subsequent advances are reviewed in Refs. 80-83. A characteristic feature of these advances is that increasing attention is given to the differential relationships that describe the development of various correlations. These relationships make it possible to include many more details of the physics of the problem, in contrast with earlier theories, which tended to lump much of the detail into such quantities as a mixing length or an eddy viscosity coefficient.

In particular, increased attention has recently been given to problems where the turbulence is expected to develop anisotropy, a condition often encountered in swirling flows. Generalized models of the turbulence that allow for anisotropy have been developed.[84-88] These anisotropic effects are embedded in a theory which makes no allowance for asymmetric states of stress; thus, the shear-stress tensor is taken to be symmetric, (e.g., $\tau_{12} = \tau_{21}$), but is allowed to have unequal elements (e.g., $\tau_{12} \neq \tau_{13}$).

The suggestion has been made,[89-95] however, that the anisotropy of the turbulence field provides a mechanism for organized angular momentum exchange, and that, consequently, it is incorrect to admit anisotropic Reynolds stresses without also making provision for their asymmetry, and for the addition of "Reynolds couples" as well. In these papers, the turbulence field is visualized as consisting of a distribution of eddies whose angular-velocity statistics are not completely random, but have a net bias toward a preferred direction in orientation space. This partially ordered array of eddies is viewed as a microstructure, and it is argued that generalized continuum equations are required for a proper description of the motion.

To derive such a set of relations, Nikolaevskii writes the equations of motion in volume- and surface-integral form for a fluid element which is small compared to the significant scale of the problem yet large enough to contain the turbulent eddy microstructure.[92,94,96] This leads to a set of equations which are superficially identical to the conventional ones, but in which the Reynolds stress is defined as a surface integral

$$R_{ij} = -\langle \rho v_i v_j \rangle_j$$

where v denotes the fluctuating part of the velocity and the notation $\langle \ \rangle_j$ denotes an average over the surface which is normal to the j-axis. The Reynolds stress defined in this manner is not, in general, symmetric. In addition, the angular momentum intrinsic to the volume element is nonzero, and what might be termed a "Reynolds couple" tensor, in complete analogy to the equations of Sec. II, contributes to its rate of change.

Several observation are in order concerning these results. The first is to note the limitation imposed by the requirement that the volume element must be larger than the largest eddy. In most of the categories already discussed, the scale of the microstructures is independent of the scale of the flow, and there are many problems which meet the condition that the former scale must be small compared with the latter. But in turbulent flows, the largest of the eddy sizes is often comparable to the scale of the body which is producing the turbulence; the range of flow situations for which the appropriate separation of scales can be realized remains to be determined.

As a second observation, it should be stressed that the asymmetric-stress effects in Nokolaevskii's work enter solely from the averaging procedure used—they are not present in the Euler or Navier-Stokes equations from which the derivation begins, and which are taken to be sufficient for describing the deterministic flow of the fluid at a point. Because the flow contains random phenomena, it is inevitable that the deterministic picture must give way to an averaging process of some sort in which a certain amount of information about the flow is lost. Nikolaevskii emphasizes the fact that the very choice of the averaging process itself may contain a tacit assumption about stress asymmetry beyond that already implicit in the Navier-Stokes equations. There is a very close parallel in Brenner's theory of suspensions,[52] where the flow over a single particle is adequately described by the Navier-Stokes equations, but where the average stress acting on a volume containing a large number of these particles may be asymmetric.

Nikolaevskii's work must be distinguished from those which treat the turbulent flow of a micropolar fluid[97] (i.e., a fluid which displays stress asymmetry at the microscopic level, and hence is not described by the Navier-Stokes equations), and from those which simply propose the asymmetric-stress equations as a model of turbulence[98,99] without attempting any derivation from more basic principles.

It should also be mentioned that Nikolaevskii[91] and Beran[100] have called attention to the fact that Reynolds in his original papers drew a careful distinction between spatial and temporal averages, and recognized the possibility that the stress tensor might be asymmetric.

The extent to which the introduction of asymmetric stress may be of value in explaining turbulent-flow phenomena remains to be determined by a detailed working out of its theoretical predictions, and their comparison with properly selected experiments. The writer's view is that the applicability of this approach must continue to be examined in connection with such areas as the preferred angular velocities of turbulent structures in shear layers,[10] the aircraft trailing vortex, and with certain problems in geophysical fluid dynamics, where the rotation of the earth can lead to anisotropic turbulent diffusion,[102] and to phenomena which are often described in terms of a negative viscosity.[103]

IV. Conclusions

To place this discussion in perspective, it must be repeated that orientational effects are not of major importance in most of the situations of interest in aeronautical and aerospace engineering. These situations usually involve the motion of dilute gases in response to gradients whose magnitudes are quite modest compared with what is required for the generation of significant orientation. In the vast majority of cases, the asymmetric stresses, if they occur at all, do so with very small amplitude, and are confined to strong-gradient regions of very limited extent. Thus, they have a neglible effect on lift, drag, vehicle performance, or any of the ordinary concerns of aerospace engineering. For these applications, the approximations inherent in the conventional equations of motion are sufficiently accurate.

Moreover, it must also be stressed that the potential of the conventional equations is still far from being exhausted. Even without any orientational effects, the conventional equations of motion are quite difficult to solve, for many practical cases. Thus, in attempting to match theory and experiment, one's first instinct must be to question whether the conventional theory has been properly formulated, its solution properly found, and all its implications properly interpreted. The existence of asymmetric states of stress should not be considered an instant remedy for unsolved problems; in virtually all aerospace engineering situations, the remedy is more likely to be found in further exploitation of the conventional approaches.

Nonetheless, there must also be a continued awareness that the conventional approximations ignore orientational effects entirely, and that situations can arise where these effects must be taken into consideration. The need for this awareness has increased in recent years, when more and more attention has been diverted away from aerospace problems and toward such areas as water purification, atmospheric turbulence, and biomedical fluid mechanics. The sampling of problems listed in Sec. III was chosen in an attempt to identify some of the areas where one must be especially alert to the possible occurrence of asymmetric states of stress.

Many of the applications mentioned here might be labeled "passive," in the sense that the main value of the new theory is seen in its ability to explain phenomena already observed. Future studies will undoubtedly see more "active" applications of the theory, where the orientational effects would be exploited. For example, the measurement of particle alignment in low-density flows, described in Sec. III, suggests a new diagnostic technique for the probing of shock-wave structure in gases. There are probably many other instances where the occurrence of oriented flows could be used to good advantage.

Asymmetric states of stress are obviously a very widespread phenomenon, present in situations as diverse as the flow of blood and the movement of weather patterns. Yet this wide range of examples is united by a common thread, and described by essentially the same set of equations. It is hoped that this review will generate a deeper appreciation of the approximations made in neglecting asymmetric stress, and a broader perspective toward problems where this approximation may no longer be acceptable.

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